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CALLAN-SYMANZIK AND RENORMALIZATION GROUP EQUATION IN THEORIES WITH SPONTANEOUSLY BROKEN SYMMETRY*

ELISABETH KRAUS

Physikalisches Institut, Universität Bonn
Nußallee 12, D-53115 Bonn, Germany

Abstract

Callan-Symanzik and renormalization group equation are discussed for the $U(1)$-axial model and it is shown, that the symmetric model is not the asymptotic version of the spontaneously broken one due to mass logarithms in the β-functions. The Callan-Symanzik equation of the standard model is seen to have the same form as the one of the simple model.

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CALLAN-SYMANZIK AND RENORMALIZATION GROUP EQUATION IN THEORIES WITH SPONTANEOUSLY BROKEN SYMMETRY

ELISABETH KRAUS*
Universität Bonn, Nußallee 12
D-53115 Bonn, Germany
E-mail: kraus@avzw02.physik.uni-bonn.de

ABSTRACT

CS and RG equation are discussed for the $U(1)$-axial model and it is shown, that the symmetric model is not the asymptotic version of the spontaneously broken one due to mass logarithms in the β-functions. The CS-equation of the standard model is seen to have the same form as the one of the simple model.

1. Introduction

All perturbative calculations suffer from breaking off at a finite, even low, order of loops, whereas the perturbative expansion is an infinite power series in the couplings. In order to be able to sum up higher order large contributions, one can use renormalization group (RG) invariance and the Callan–Symanzik (CS) equation, which have been successfully applied in symmetric models as it is QED. However, if the theory is spontaneously broken, RG invariance can be formulated only for the coupling, which is the perturbative expansion parameter. But the CS equation, which describes the breaking of dilatations, involves all interactions which are differently renormalized and contains therefore the β-functions of mass parameters. We will show, that in the spontaneously broken model, the β-functions depend logarithmically on the mass parameters, also in the asymptotic region, from 2-loop order onwards. The symmetric massless model is therefore not reached in the asymptotic limit. In the last section we give the CS equation of the standard model and the 1-loop β-functions of the gauge interactions in the on-shell scheme. It is seen, that it involves mass β-functions in 1-loop order. Therefore an analysis of the symmetric massless $SU(2) \times U(1)$ theory is not sufficient for determining leading contributions of higher orders.

2. The spontaneously broken $U(1)$-axial model

The $U(1)$-axial model is the simplest model with two different masses, which are generated by the spontaneous symmetry breaking. It can be considered as a toy model of the matter sector of the standard model, when the gauge interactions are turned off. The model describes the interaction of a massive scalar H, a massless pseudoscalar χ and one massive fermion ψ. The classical action of the model is given

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by:

$$\Gamma_{cl} = \int\Big(\frac{1}{2}(\partial H\partial H + \partial\chi\partial\chi) + i\bar\psi\partial\!\!\!/\psi - m_f\bar\psi\psi - \frac{m_f}{m_H}\sqrt{\frac{\lambda}{3}}\bar\psi(H + i\gamma_5\chi)\psi$$
$$-\frac{1}{2}m_H^2H^2 - \frac{1}{2}m_H\sqrt{\frac{\lambda}{3}}H(H^2+\chi^2) - \frac{\lambda}{4!}(H^2+\chi^2)^2\Big) \tag{1}$$

As indicated by the notation we will consider the model throughout in the on-shell parametrization, where the masses of the particles are fixed at the poles of the respective propagators. The invariance of the classical action under the axial $U(1)$-transformations can be expressed in terms of a Ward identity:

$$\mathbf{W}\Gamma = 0 \quad \text{with} \quad \mathbf{W} = -i\int((H+v)\frac{\delta}{\delta\chi} - \chi\frac{\delta}{\delta H} - \frac{i}{2}\frac{\delta}{\delta\psi}\gamma_5\psi - \frac{i}{2}\bar\psi\gamma_5\frac{\delta}{\delta\bar\psi}) \tag{2}$$

The Ward identity (2) together with appropriate normalization conditions uniquely determine the Green functions to all orders of perturbation theory.

$$\begin{array}{lll} \partial_{p^2}\Gamma_{HH}\big|_{p^2=\kappa^2} = 1 & \gamma^\mu\partial_{p^\mu}\Gamma_{\psi\bar\psi}\big|_{p\!\!\!/=\kappa} = 1 & \Gamma_H = 0 \\ \Gamma_{HH}\big|_{p^2=m_H^2} = 0 & \Gamma_{\bar\psi\psi}\big|_{p\!\!\!/=m_f} = 0 & \Gamma_{HHHH}\big|_{p^2=\kappa^2} = -\lambda \end{array} \tag{3}$$

We have fixed the residua and the coupling at an Euclidean normalization point κ^2. The 4-point function of the Higgs interaction is evaluated at a symmetric momentum p^2 defined by

$$p_i^2 = p^2\,, \qquad p_i\cdot p_j = -\tfrac{2}{3}p^2 \quad i\neq j\,. \tag{4}$$

With these normalization conditions the Green functions are calculated perturbatively in powers of the coupling $\sqrt{\lambda}$, the shift parameter is determined by the Ward identity:

$$v \equiv v(m_H, m_f, \kappa, \lambda) = \sqrt{\frac{3}{\lambda}}m_H + O(\hbar) \tag{5}$$

The CS equation describes the breaking of the dilatations for the off-shell Green functions in form of a differential equation[1]. In the tree approximation dilatations are broken by the mass terms, this breaking is related to the differentiation with respect to the Higgs due to the spontaneous symmetry mechanism and is determined by the covariance with respect to the Ward identity (2):

$$m\partial_m\Gamma_{cl} = \int\Big(v\frac{\delta\Gamma_{cl}}{\delta H} + \frac{m_H^2}{2}(H^2+\chi^2+2vH)\Big) \tag{6}$$

$m\partial_m$ is the dilatational operator when acting on the Green functions

$$m\partial_m \equiv m_H\partial_{m_H} + m_f\partial_{m_f} + \kappa\partial_\kappa \tag{7}$$

In higher orders dilatations are broken by anomalies which are given by the β-functions and anomalous dimensions. The CS equation is derived to all orders of perturbation theory[2]:

$$(m\partial_m + \beta_{m_f} m_f \partial_{m_f} + \beta_\lambda \partial_\lambda - \gamma_S \mathcal{N}_S - \gamma_F \mathcal{N}_F)\Gamma = v(1+\rho)\int (\frac{\delta}{\delta H} + \hat{\alpha}\frac{\delta}{\delta q})\Gamma\big|_{q=0} \quad (8)$$

$\mathcal{N}_S$ and $\mathcal{N}_F$ are the leg counting operators of the scalars and fermions respectively. The r.h.s. is the unique continuation of (6) to higher orders, where q is an external field coupled to the invariant of UV dimension 2.

The β-function with respect to the mass parameter is special for models with spontaneous breaking of the symmetry. Due to the fact that the mass differentiation acts not only on the vertices but also on propagators it brings about, that mass logarithms appear in the β-functions of the CS equation from 2-loop order onwards. Finally we want to mention that the r.h.s. of the CS equation potentially contains non-integrable infrared divergencies, which have to be shown to be absent by an explicit test on the 2-point function of the massless particles.

The RG equation describes the response to a change of the arbitrary normalization point κ. Due to the physical normalization conditions it has the simple form

$$\Big(\kappa\partial_\kappa + \tilde{\beta}_\lambda \partial_\lambda - \tilde{\gamma}_S \mathcal{N}_S - \tilde{\gamma}_F \mathcal{N}_F\Big)\Gamma = 0 \quad (9)$$

with the additional constraint:

$$\kappa\partial_\kappa v + \tilde{\beta}_\lambda \partial_\lambda v + \tilde{\gamma}_S v = 0 \quad (10)$$

One has to note that the CS and RG equation do not contain the same differential operators and that the β-functions and anomalous dimensions are not equal in general. In order to get insight into mass parameter dependence in higher orders they have to be solved consistently at the same time. For this purpose we want to consider in the next section the consequences of RG invariance.

3. Renormalization group invariance

In order to simplify the analysis we want to restrict the considerations of RG invariance to the invariant charge, which is defined in the $U(1)$-axial model by the following combinations of Green functions evaluated at the symmetric point p^2 (4):

$$Q(\tfrac{p^2}{\kappa^2}, \tfrac{m^2}{p^2}, \lambda) = -\Gamma_4(p^2, m^2, \kappa^2, \lambda)\Big(\partial_{p^2}\Gamma_2(p^2, m^2, \kappa^2, \lambda)\Big)^{-2} \quad (11)$$

It satisfies the homogeneous RG equation and is therefore a RG invariant

$$(\kappa\partial_\kappa + \tilde{\beta}_\lambda \partial_\lambda) Q(\tfrac{p^2}{\kappa^2}, \tfrac{m^2}{p^2}, \lambda) = 0\,. \quad (12)$$

According to (3) the invariant charge has well-defined normalization properties

$$Q(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2},\lambda)\Big|_{p^2=\kappa^2}=\lambda. \tag{13}$$

Finite RG transformation of the invariant charge can be derived by a formal integration of the RG equation[3], but on a much more fundamental level, invariance of the Green functions under finite RG transformations up to field redefinitions can be postulated directly[4,5]: If one fixes the invariant charge at a different normalization point κ'^2 to a different coupling λ'

$$Q(\tfrac{p^2}{\kappa'^2},\tfrac{m^2}{p^2},\lambda')\Big|_{p^2=\kappa'^2}=\lambda' \tag{14}$$

RG invariance requires that for all momenta the result has to be the same:

$$Q(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2},\lambda)=Q(\tfrac{p^2}{\kappa'^2},\tfrac{m^2}{p^2},\lambda') \tag{15}$$

Therefrom one derives the multiplication law of the RG

$$Q(\frac{p^2}{\kappa^2},\frac{m^2}{p^2},\lambda)=Q(\frac{p^2}{\kappa'^2},\frac{m^2}{p^2},Q(\frac{\kappa'^2}{\kappa^2},\frac{m^2}{\kappa'^2},\lambda)) \tag{16}$$

Restricting to perturbation theory, where the invariant charge is calculated in powers of the coupling λ,

$$Q(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2},\lambda)=\sum_{i=0}^{\infty}\lambda^{i+1}f_i(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2})\,, \tag{17}$$

one is able to solve the multiplication law of RG order by order in perturbation theory[6]:

$$\begin{aligned}
f_1(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2}) &= g_1(\tfrac{m^2}{\kappa^2})-g_1(\tfrac{m^2}{p^2})\\
f_2(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2}) &= g_2(\tfrac{m^2}{\kappa^2})-g_2(\tfrac{m^2}{p^2})+(g_1(\tfrac{m^2}{\kappa^2})-g_1(\tfrac{m^2}{p^2}))^2\\
f_3(\tfrac{p^2}{\kappa^2},\tfrac{m^2}{p^2}) &= g_3(\tfrac{m^2}{\kappa^2})-g_3(\tfrac{m^2}{p^2})+\frac{5}{2}(g_2(\tfrac{m^2}{\kappa^2})-g_2(\tfrac{m^2}{p^2}))(g_1(\tfrac{m^2}{\kappa^2})-g_1(\tfrac{m^2}{p^2}))\\
&\quad+\frac{1}{2}(g_2(\tfrac{m^2}{\kappa^2})g_1(\tfrac{m^2}{p^2})-g_2(\tfrac{m^2}{p^2})g_1(\tfrac{m^2}{\kappa^2}))+(g_1(\tfrac{m^2}{\kappa^2})-g_1(\tfrac{m^2}{p^2}))^3
\end{aligned} \tag{18}$$

Therefrom it is seen that RG invariance structures the invariant charge according to its mass and momentum dependence. If the functions $g_i(u)$ are not constant lower order functions are induced to higher orders and RG invariance can only be realized to all orders of perturbation theory.

From RG invariance it is not possible to gain information about the functions $g_i(u)$. They have to be calculated in perturbation theory by the respective loop diagrams. However, because one has derived the CS equation, it is possible to find restrictions on the high energy behavior of these functions. Considering theories

without spontaneous symmetry breaking, as it is ϕ^4 and QED, the CS equation involves the same differential operators as the RG equation (12):

$$(m\partial_m + \beta_\lambda\partial_\lambda)Q(\tfrac{p^2}{\kappa^2}, \tfrac{m^2}{p^2}, \lambda) = Q_m(\tfrac{p^2}{\kappa^2}, \tfrac{m^2}{p^2}, \lambda) \tag{19}$$

$Q_m(\frac{p^2}{\kappa^2}, \frac{m^2}{p^2}, \lambda)$ denotes a soft insertion, which vanishes for asymptotic non-exceptional momenta. Applying the CS equation to the invariant charge as it is given by RG invariance (18) one finds that all the functions $g_i(u)$ tend to logarithms in the asymptotic region:

$$g_i(\tfrac{m^2}{p^2}) \longrightarrow \frac{1}{2} b_i \ln |\tfrac{m^2}{p^2}| \quad \text{for} \quad p^2 \to -\infty \tag{20}$$

Moreover one can show that for an asymptotic normalization point κ^2 the massless limit is reached smoothly, especially the β-functions of the RG and CS equation are equal to all orders of perturbation theory and mass independent[7]:

$$\beta_\lambda = \tilde{\beta}_\lambda \quad \text{for} \quad \kappa^2 \to -\infty \tag{21}$$

These are the intrinsic normalization conditions of the MS- and $\overline{\text{MS}}$-scheme.

4. RG invariance and CS equation in the $U(1)$-axial model

In order to get insight into the specific structure of the spontaneously broken models we consider the invariant charge of the Higgs coupling as it is defined in (11). It satisfies the homogeneous RG equation (12) and for an asymptotic symmetric momentum the CS equation (8)

$$(m\partial_m + \beta_{m_f} m_f \partial_{m_f} + \beta_\lambda\partial_\lambda)Q(\tfrac{p^2}{\kappa^2}, \tfrac{m^2}{p^2}, \lambda) \longrightarrow 0 \quad \text{for} \quad p^2 \to -\infty\,. \tag{22}$$

As a further simplification we take the normalization point, which normalizes the coupling and the residua, also in the asymptotic region. Such a choice corresponds to an on-shell definition for the masses and a MS definition for residua and the coupling. It can be shown that for asymptotic momenta the β-functions of the coupling in the RG and CS equation are equal

$$\beta_\lambda = \tilde{\beta}_\lambda \quad \text{for} \quad \kappa^2 \to -\infty \tag{23}$$

In 1-loop order the β-functions are calculated to be

$$\begin{aligned}
\beta^{(1)}_\lambda &= \frac{1}{8\pi^2}\frac{1}{3}(5 - 8\frac{m_f^4}{m_H^4} + 4\frac{m_f^2}{m_H^2})\lambda^2 \equiv b^{(1)}_\lambda(\frac{m_f}{m_H})\lambda^2 \\
\beta^{(1)}_{m_f} &= -\frac{1}{16\pi^2}\frac{1}{3}\Big(5 - 8\frac{m_f^4}{m_H^4}\Big)\lambda \equiv b^{(1)}_{m_f}(\frac{m_f}{m_H})\lambda
\end{aligned} \tag{24}$$

For solving the CS and RG equation consistently their commutators have to vanish which gives the following restrictions on the β-functions:

$$\kappa\partial_\kappa\beta_\lambda = \beta_{m_f} m_f \partial_{m_f}\beta_\lambda \qquad \kappa\partial_\kappa\beta_{m_f} = \beta_\lambda\partial_\lambda\beta_{m_f} \tag{25}$$

Therefrom it is seen that the β-functions logarithmically depend on the normalization

point from 2-loop order onwards. For the 2-loop functions one obtains the result[6]:

$$\begin{aligned}
\beta^{(2)}_\lambda &= \lambda^3\Big(-\frac{1}{2}b^{(1)}_{m_f}(\tfrac{m_f}{m_H})m_f\partial_{m_f}b^{(1)}_\lambda(\tfrac{m_f}{m_H})\ln|\frac{m_H^2}{\kappa^2}| + b^{(2)}_\lambda(\tfrac{m_f}{m_H})\Big)\\
\beta^{(2)}_{m_f} &= \lambda^2\Big(\frac{1}{2}b^{(1)}_{m_f}(\tfrac{m_f}{m_H})b^{(1)}_\lambda(\tfrac{m_f}{m_H})\ln|\frac{m_H^2}{\kappa^2}| + b^{(2)}_{m_f}(\tfrac{m_f}{m_H})\Big)
\end{aligned} \tag{26}$$

$b^{(2)}_\lambda$ and $b^{(2)}_{m_f}$ are normalization point independent 2-loop contributions. The invariant charge can be constructed as the simultaneous solution of the CS and RG equation, if one takes into account the mass dependent 1-loop induced contributions of the consistency equation. Inserting in the RG invariant solution (18) one gets in 1- and 2-loop order for the invariant charge at asymptotic p^2 and κ^2:

$$\begin{aligned}
Q^{(1)}_{as} &= \frac{1}{2}\lambda^2 b^{(1)}_\lambda(\tfrac{m_f}{m_H})\ln(\tfrac{p^2}{\kappa^2})\\
Q^{(2)}_{as} &= \lambda^3\Big(\Big(\frac{1}{2}b^{(1)}_\lambda(\tfrac{m_f}{m_H})\ln(\tfrac{p^2}{\kappa^2})\Big)^2 + \frac{1}{2}b^{(2)}_\lambda(\tfrac{m_f}{m_H})\ln(\tfrac{p^2}{\kappa^2})\\
&\quad -\frac{1}{8}b^{(1)}_{m_f}(\tfrac{m_f}{m_H})m_f\partial_{m_f}b^{(1)}_\lambda(\tfrac{m_f}{m_H})\Big(\ln^2|\tfrac{m_H^2}{\kappa^2}| - \ln^2|\tfrac{m_H^2}{p^2}|\Big)\Big)
\end{aligned} \tag{27}$$

This shows that the 2-loop RG invariant contains quadratic logarithms induced by the CS equation. They appear due to the fact that the fermion mass has to be treated as an independent parameter of the model, being differently renormalized when compared to the coupling constant of the Higgs 4-point interaction. RG invariance can be only formulated for the coupling λ and does not include the fermion mass in order to be able to construct the S-matrix of the theory. The asymptotic behavior of the spontaneously broken model differs from the symmetric theory by these mass dependent logarithms. As a consequence an asymptotic limit in the sense of mass independence does not exist: The asymptotic normalization conditions are defined by the requirement that the terms of order $m^2/\kappa^2\ln|m^2/\kappa^2|$ can be neglected. The smaller these terms are chosen the larger the mass dependent logarithms in the β-functions will grow. Therefore the massless symmetric theory is not the asymptotic version of the spontaneously broken one.

5. The Callan–Symanzik equation of the standard model

In the standard model of electroweak interactions the masses of all massive particles are generated by the spontaneous breaking of $SU(2)\times U(1)$ gauge invariance. The massive gauge bosons are the charged bosons $W_\pm$ with mass M_W and the neutral boson Z with mass M_Z. The minimal standard model contains a complex scalar doublet Φ with three unphysical would-be Goldstones and the physical field H with mass m_H. In addition it involves also the electromagnetic interactions coupled to the massless photon field A_μ. In the fermionic sector there are the left handed lepton

and quark doublets and right handed singlets. For simplification we do not consider mixing between different families and, especially, assume CP-invariance.

In order to have free field propagators with a good ultraviolet and infrared behavior one uses the linear R_ξ-gauges breaking thereby $SU(2)\times U(1)$ gauge invariance and also rigid invariance. Gauge invariance has to be replaced by BRS invariance introducing the Faddeev–Popov ghosts c_a and $\bar{c}_a, a = +, -, Z, A$. The Green functions have to be constructed in accordance with the Slavnov–Taylor (ST) identity, which is the functional version of BRS invariance. It turns out that the Green functions are not completely determined by the ST identity – in addition one has to use the Ward identity of rigid invariance and the remaining $U(1)$-local Ward identity in order to fix the charges of the fermions. Using an on-shell scheme for determining the free parameters, one has one coupling, which can be fixed in the Thompson limit to be the fine structure constant $\alpha = e^2/4\pi$.

$$M_W,\, M_Z,\, m_H,\, m_f,\, e \tag{28}$$

are the free parameters of the standard model, which are complemented by two gauge parameters and the masses of the ghosts

$$\xi, \hat{\xi}, \zeta_W M_W, \zeta_Z M_Z \tag{29}$$

With this choice of parameters a normalization point κ^2 is only introduced for fixing infrared divergent residua off-shell and the RG equation is trivial concerning the coupling.

The CS equation of the standard model gives the breaking of dilatations in form of a differential equation the same way as in the $U(1)$-axial model. In the tree approximation dilatations are broken by the mass terms of the fields They are related to the differentiation with respect to the Higgs field and to the external Higgs field which had to be introduced for establishing the Ward identity of rigid symmetry:

$$m\partial_m \Gamma_{cl} = \int v\Big(\frac{\delta\Gamma_{cl}}{\delta H} + \zeta\frac{\delta\Gamma_{cl}}{\delta\hat{H}}\Big) + \frac{m_H^2}{2}\Delta_{inv} \tag{30}$$

Δ_{inv} is the 2-dimensional BRS and rigid invariant scalar polynomial. The r.h.s. potentially contains non-integrable infrared divergencies, which have to be proven to be absent. Therefore it is necessary to have propagators which are well-behaving in the infrared, i.e. one has to use the physical fields and has to impose to all orders that the mixing between massless and massive fields vanishes at $p^2 = 0$. In the ghost sector this requires to introduce an independent angle θ_G defined by the mass ratio of the ghosts:

$$1 + \tan\theta_W \tan\theta_G \equiv \frac{\zeta_Z M_Z^2}{\zeta_W M_W^2} \qquad \cos\theta_W \equiv \frac{M_W}{M_Z} \tag{31}$$

The CS equation is uniquely determined if one uses invariance of the Green functions with respect to the ST identity, rigid symmetry and $U(1)$-gauge symmetry. For

simplification we neglect all the contributions which involve Faddeev–Popov ghosts and external fields. In 1-loop order the CS equation then reads[8]:

$$
\begin{aligned}
&\Big\{ m\partial_m + \beta_e e\partial_e + \beta_{m_H} m_H \partial_{m_H} - \beta_{M_W}\Big(\partial_{\theta_W} - \int (Z^\mu \frac{\delta}{\delta A^\mu} - A^\mu \frac{\delta}{\delta Z^\mu})\Big) \\
&-\hat\gamma_V\Big(\int(\sin\theta_W Z^\mu + \cos\theta_W A^\mu)(\sin\theta_W \frac{\delta}{\delta Z^\mu} + \cos\theta_W \frac{\delta}{\delta A^\mu}) + 2(\hat\xi + \xi)\partial_{\hat\xi}\Big) \\
&-\gamma_V\Big(\mathcal{N}_V + 2\xi(\partial_\xi + \partial_{\hat\xi}) + \sin\theta_G\cos\theta_G\partial_{\theta_G}\Big) - \gamma_S \mathcal{N}_S \\
&+\sum_{f_i}(\beta_{m_{f_i}} m_{f_i}\partial_{m_{f_i}} - \gamma^R_{f_i}\mathcal{N}_{f_i}) - \sum_{F_i}\gamma^L_{F_i}\mathcal{N}_{F_i}\Big\}\Gamma\Big|_{\substack{ext.f.=0 \\ c_a,\bar c_a=0}} = [\Delta_s]^3_3 \cdot \Gamma
\end{aligned}
\qquad (32)
$$

There $[\Delta_s]^3_3$ denotes the soft insertion, which uniquely continues the classical soft breaking of dilatations to higher orders. The operators $\mathcal{N}_A$ are the usual leg counting operators of vectors, scalars, right handed fermions and left handed fermion doublets. The β-functions of the electromagnetic coupling and the of the mass ratio $\frac{M_W}{M_Z}$ are calculated to

$$
\begin{aligned}
\beta_e &= -\frac{\alpha}{24\pi}\Big(42 - \tfrac{64}{3}N_F\Big) \\
\beta_{M_W} &= -\frac{\alpha}{24\sin\theta_W\cos\theta_W}\Big((43 - 8N_F) - (42 - \tfrac{64}{3}N_F)\sin^2\theta_W\Big)
\end{aligned}
\qquad (33)
$$

One has to note that the CS equation of the standard model has unusual ingredients compared to the symmetric theory: It involves the mixed field differentiation operators due to the fact the unbroken electromagnetic symmetry has also SU(2) interactions. Especially there appear the β-functions with respect to the different masses of the standard model. Because these β-functions are mass dependent already in one loop order, the 2-loop β-functions depend on mass logarithms in the same way as we have derived in the $U(1)$-axial model.

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